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NUMERICAL SIMULATIONS OF SHOCKLESS NONLINEAR ACOUSTICS NOISE IN ONE DIMENSION

by

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NUMERICAL SIMULATIONS OF SHOCKLESS NONLINEAR ACOUSTIC NOISE IN ONE DIMENSION

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ABSTRACT

The attenuation of a monochromatic signal in the presence of discrete noise in one dimension is investigated numerically. The predicted Gaussian attenuation is verified by the numerical program, which is based on Riemann's implicit solution of the exact equation for the unidirectional propagation of shockless sound. Two new results are also presented. In the first, the transition from Gaussian to Bessel dependence as a function of resolution in the detection of a signal is observed. This results shows that the fundamental property of time reversibility can only be established if the overall system of the waves and the observer is considered. In the second result, the evolution of the amplitude of a signal injected downstream from the noise is investigated. The Gaussian attenuation is also observed in this case. This result explicitly shows that the attenuation length depends on the distance the signal has traveled, thus displaying memory and breakdown of translational invariance.

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I. INTRODUCTION

In the presence of high intensity broad band noise, a small amplitude signal attenuates due to nonlinear interactions with the noise. If the noise is isotropic, the amplitude attenuates as an exponential. This result was first predicted by Landau and Rumer (1937) in connection with ultrasonic attenuation due to thermal phonons in a dielectric at low temperature. Westervelt (1976) extended these ideas to the case where the broad band noise was due to classical sound waves. Experimental observations of the attenuation of sound in superfluid helium (Maris, 1973) and of sound in water (Stanton and Beyer, 1978 and 1981) support these results.

The limiting case of interaction with noise in one dimension is derived from Riemann's exact equation for lossless propagation of unidirectional sound

$$u_t + (c+\beta u)u_x = 0 \ , \qquad (1.1)$$

where u is the particle velocity, $\beta = 1 + (\rho/c)(dc/d\rho)$ is the nonlinear coefficient, c is the equilibrium value of the speed of sound, and ρ is the ambient density. The amplitude of the signal is calculated to attenuate as a *Gaussian* $\exp(-\Gamma x^2)$, where x is the distance from the source, and where the nonlinear attenuation coefficient is

$$\Gamma = \frac{2\pi^2\beta^2 f_o^2}{\rho c^5}\int_0^\infty \mathbf{I}_f \, df \quad , \qquad (1.2)$$

where f_0 is the frequency of the signal and $\mathbf{I}_f$ is the spectral intensity of the noise at frequency f (Rudenko and Chirkin, 1975; Rudenko and Soluyan, 1977). The integral is the intensity of the noise. Hence, in the shockless regime β is independent of the details of noise spectral distribution. It should be noted that this is consistent with the $\exp(-\Gamma x^2)$ solution to the evolution equation $dA/dx = -2\Gamma x A$ for the amplitude A. If Γ were dependent upon the distribution, then the spectral evolution of the noise due to nonlinear interactions would necessarily cause Γ to be a function of position, which would invalidate the Gaussian solution.

The basic nonlinear interaction of acoustic waves is a three-wave resonance, so that waves with frequency f_1 and f_2 scatter to produce waves with frequencies $f_3 = |f_1 \pm f_2|$. Furthermore, because these waves are nondispersive the interactions can only be collinear (Rudenko and Soluyan, 1977). For random waves in two and three dimensions, energy in the noise components redistributes irreversibly by collinear resonant interactions. Randomization in angle is brought about by slower collision processes with a local transfer between adjacent rays (Newell and Aucoin, 1971). After a time determined by the nonlinearities, a closed system of interacting random waves can thus reach thermodynamic equilibrium, and small fluctuations about this state relax exponentially with an attenuation coefficient proportional to the noise intensity. Because the noise effectively acts as a heat bath, it is appropriate to refer to the *absorption* of sound by noise in this case.

The predicted Gaussian attenuation in one-dimensional nonlinear acoustics brings together two physically significant results. First, because a small amplitude signal can be regarded as a fluctuation of the noise, and because this fluctuation does not relax exponentially, we infer that a system of interacting, collinear, nondispersive waves is inherently far off equilibrium. This results from both a lack of angle randomization and the fact that all spectral components are in resonance with each other in this case.

A second aspect of a Gaussian attenuation is that it breaks translational invariance because the attenuation coefficient is now a function of position. Specifically, by taking measurements at several distances from the source of a signal, one can calculate both the location and strength of the source. This cannot be done if the attenuation is exponential. This is physically significant because the equations that describe the evolution of nonlinear noise and its interaction with a monofrequency signal are translationally invariant.

An experiment to measure the predicted Gaussian attenuation has been carefully investigated by Larraza et al. (1996). In this experiment, the dimensionality is controlled by performing measurements in a long traveling wave tube driven at frequencies below the first cutoff. The theory is in excellent agreement with the experimental results only after it is modified to incorporate wall losses. The agreement is shown as the frequency, noise level, and distance from the source are varied. In addition, Larraza et al. (1996) observed the spectral intensity of the high-frequency tail of fully developed shockless noise to be an f^{-3} power law in the frequency f, in accord with Kutznetsov's theory (1970). This power law is a consequence of the far off equilibrium nature of the system.

The purpose of this thesis is to numerically investigate the predicted Gaussian attenuation. In Ch. II we give a general theoretical background for nonlinear acoustics in one dimension. In Ch. III we describe the details of the numerical code. The program is tested by comparing the numerical results with the predictions of theory for the cases of pure tone propagation and suppression of a signal by a pump. In Ch. IV we present numerical simulations of the absorption of sound by noise. We verify the absorption of sound by noise and also present two new results. In the first, we investigate the transition from Gaussian to Bessel as a function of resolution in the detection of a signal in the presence of noise. In the second result, as a test of the breakdown of translational invariance, we investigate the evolution of the amplitude of a signal injected downstream in the presence of noise. Conclusions and possible future work are presented in Ch. V.

II THEORY

In this chapter, we describe Riemann's equation for sound propagation in one dimension in a barotropic fluid, and then derive the general shock inception distance. We also present the analytic solutions to some special cases. The simplicity of Riemann's solution allows the determination of exact and asymptotic explicit solutions in the preshock region. Examples include Fubini's (1935) pure tone radiation problem, Fenlon's (1970) generalization to multiple frequency sinusoidal source, Kutznetsov's (1970) asymptotic f^{-3} spectrum for broadband noise, and Rudenko and Chirkin's (1975) Gaussian attenuation of a signal due to interactions with normally distributed random noise.

A. RIEMANN'S EQUATION

The basic equations of motion for an isentropic perfect gas are governed by the continuity equation and Newton's second law

$$\rho_t + u\rho_x + \rho u_x = 0 \ , \qquad (2.1a)$$

$$u_t + uu_x + \rho^{-1}p_x = 0 \ , \qquad (2.1b)$$

respectively, where ρ is density, p is total pressure, and u is the particle velocity. The subscripts x and t denote partial differentiation with respect to the space and time coordinates, respectively. Eqs. (2.1) must be supplemented with the equation of state of an isentropic ideal gas

$$\frac{p}{p_o} = \left(\frac{\rho}{\rho_o}\right)^{\gamma} , \qquad (2.2)$$

where ρ_0 and p_0 are the ambient density and pressure, respectively, and γ is the ratio of specific heats of the gas.

For the situation where the waves are propagating in one direction, Eqs. (2.1) and (2.2) imply Riemann's equation (Blackstock, 1972)

$$u_t + (c + \beta u)u_x = 0 \ , \tag{2.3}$$

where $c = (\partial p/\partial \rho)_0$ is the equilibrium value of the speed of sound, and where for an ideal gas, the nonlinear coefficient is given by $\beta = (1+\gamma)/2$. From Eq. (2.3), the instantaneous speed of propagation of a point in the wave with particle velocity u is $c+\beta u$. Thus, points of the wave having different particle velocity propagate at different speeds.

A general implicit solution of Eq. (2.4) with the $x = 0$ boundary condition $u(0,t) = f(t)$ is given by

$$u(x,t) = f\left(t - \frac{x}{c + \beta u(x,t)}\right), \tag{2.4}$$

as can be verified by direct substitution into Eq (2.3). Thus the velocity at the boundary $x = 0$ determines the velocity at $x > 0$ through the retarded time

$$\tau = t - \frac{x}{c + \beta u} \ . \tag{2.5}$$

B. SHOCK INCEPTION DISTANCE

Suppose u_1 is produced at $x = 0$ at time t_1, and a greater velocity u_2 is produced at a later time t_2. Riemann's equation breaks down when the second disturbance overtakes the first, which will eventually occur at some $x > 0$. This overtaking yields a discontinuity (infinite slope or shock) in u. If t_1 and t_2 differ by a finite amount, a discontinuity will occur at a distance less than x, because the waveform u(x,t) continuously distorts as time progresses. We are thus led to consider velocity disturbances u and u+du and at $x = 0$ that are an infinitesimal time dt apart. If $du > 0$, the disturbances will coincide at some distance L. If T is the elapsed time,

$$[c + \beta u]T = L \ , \tag{2.6a}$$

$$[c + \beta(u + du)](T - dt) = L \ , \tag{2.6b}$$

corresponding to the first and second disturbances, respectively. Multiplying out Eq. (2.6b), substituting Eq. (2.6a), and solving for L, yields

$$L = \frac{(c + \beta f)^2}{\beta\, df / dt} \ . \tag{2.7}$$

The quantities f and df/dt are the values of the velocity and its slope at $x = 0$, respectively. The expression (2.7) gives the distance at which an initial positive slope becomes infinite. (A negative slope never becomes infinite.) For any solution to Riemann's equation to be valid at some distance x, it must be verified that $x < L$ for all values of f(t).

C. PURE TONE PROPAGATION

For the propagation of a pure tone, the solution (2.4) subject to the $x = 0$

boundary condition

$$u(0,t) = u_0 \sin(\omega t) \ , \tag{2.8}$$

may be written in the form of Fourier series

$$u = u_0 \sum_{n=1}^{\infty} B_n \sin(n\omega t) \ , \tag{2.9}$$

where in the preshock region the Fourier coefficients are given by (Fubini, 1935)

$$B_n = \frac{2J_n(\beta u_0 \omega x / c^2)}{n\beta u_0 \omega x / c^2} \ , \tag{2.10}$$

where J_n is the Bessel function of order n.

For the $x = 0$ velocity (2.8), the minimum value of the shock length (2.7) corresponds to ωt being an integral multiple of 2π. The minimum distance at which shocking occurs for a pure tone is thus

$$L = \frac{c^2}{\beta u_0 \omega} \ . \tag{2.11}$$

D. SUPPRESSION OF A SIGNAL BY A PUMP

The simplest problem of interaction of sound with sound is given by the $x = 0$ boundary condition

$$u(0,t) = u_p \sin(\omega_p t) + u_s \sin(\omega_s t) \ , \tag{2.12}$$

where we focus here on the interaction of a strong pump wave of low frequency ω_p with a weak signal of high frequency ω_s. The signal has little effect on the propagation of the pump. However, the pump will modulate the signal and generate sidebands at frequencies $\omega_s \pm \omega_p$, thereby removing energy from the signal. An approximation to Fenlon's (1970) solution for the evolution of the signal is

$$u(x,t) = u_s J_o\left(\beta u_p \omega_s x / c^2\right) \sin[\omega_s (t - x/c)] \ . \tag{2.13}$$

The amplitude of the signal depends only on the particle velocity of the pump, the frequency of the signal, and the distance. The signal vanishes at the zeros of the Bessel function, at which the energy has been temporarily completely pumped into adjacent sidebands.

E. ABSORPTION BY NOISE

For the case of a weak signal of frequency f in the presence of finite-amplitude broadband noise, the amplitude of the signal attenuates as a Gaussian $\exp(-\Gamma x^2)$, where

$$\Gamma = \frac{2\pi^2 \beta^2 f^2}{c^4} u_{rms}^2 \ , \tag{2.14}$$

where u_{rms} is the rms velocity of the noise (Rudenko and Chirkin, 1975; Rudenko and Soluyan, 1977). Broadband noise may be considered by the $x = 0$ boundary condition

$$u_{noise}(0,t) = A\sum_{n=1}^{N}\cos(\omega_n t + \varphi_n) , \qquad (2.15)$$

where A is the peak velocity of each component of the noise, ω_n are densely distributed frequencies, and φ_n are randomly distributed phases. The Gaussian attenuation thus results as a limiting case of multiple pump waves with random phases. When a signal is interacting with one pump, the restitution effect (amplitude increasing with distance) is lost when the interaction occurs with a large number of pumps with random phases. Because of the central limit theorem, the noise described by (2.15) should approach a normal distributed noise for a sufficiently large number N. In this limit, a theory formulated in terms of the noise (2.15) should accurately yield the Gaussian attenuation.

By squaring Eq. (2.15) and averaging over time, we determine the square rms particle velocity of the noise. Solving for A yields

$$A = u_{rms}\sqrt{\frac{2}{N}} . \qquad (2.16)$$

The approach to the Gaussian attenuation can be probed by increasing N while keeping u_{rms} constant.

III. NUMERICAL METHOD

In this chapter, we describe the numerical method used to integrate Riemann's equation for shockless sound propagation in one dimension. We test the numerical program by comparing its results to theory for the cases of pure tone radiation and suppression of a signal by a pump.

A. FORWARD PROPAGATION METHOD

As a numerical convenience, we choose $\beta = 1$ and $c = 1$ so that the dimensionless Riemann's equation (2.3) and the expression (2.5) for the retarded time become

$$\frac{\partial u}{\partial t} + (1+u)\frac{\partial u}{\partial x} = 0, \tag{3.1}$$

and

$$\tau = t - \frac{x}{1+u}, \tag{3.2}$$

respectively. As stated in Sec. 2.1, a general implicit solution of Eq. (3.1) with the $x = 0$ boundary condition $u(0,t) = f(t)$ is given by $u(x,t) = f(\tau)$.

Eq. (3.2) determines a set of straight lines in the x-τ plane. The slope of each line is determined by the value of the velocity at the boundary at a given time (wavelet). These lines are known as *characteristics*, where the parameter τ is the time base for the source signal with $t = \tau$ at $x = 0$.

The principle of the forward propagation method stems from the fact that the characteristics represent the trajectories of motion of the different wavelets in the x-τ plane. The velocity of propagation of a wavelet is equal to 1+u where u is the assigned value of the velocity of the wavelet at the boundary. The method is implemented by storing at the boundary the wavelets and their corresponding times. In the propagation of the initial waveform the first incremental distance produces a new time array associated with the velocity array. While at the origin the spacing between time arrays is commensurate, once the waves propagate a small distance the time between adjacent elements is stretched or compressed according to Eq. (3.2), which physically corresponds to distortions of the initial waveform. Appendix A shows a C-language numerical program that implements the forward propagation method.

B. DISCRETE FOURIER TRANSFORM

Our main emphasis is in determining the evolution of the amplitude of a monochromatic signal in the presence of noise. As we forward propagate the boundary condition of a time series with the signal plus noise, sidebands of the noise appear about the signal's frequency. Also, the amplitude of the signal is expected to attenuate with distance. It is thus necessary to extract the signal from the noise at a distance x from the source. We achieve this by Fourier analyzing the waveform at the frequency of the signal, which amounts to time averaging the in-phase and quadrature components of the waveform. This process is referred to as the "discrete Fourier transform" (DFT) because the integration is performed with a finite (rather than continuous) time increment and is performed over a finite (rather than infinite) time interval.

The integration in our DFT is accomplished with trapezoidal summation. (Simpson's rule cannot be employed because the time increment is not constant.) Appendix A shows the numerical program which includes the trapezoidal summation.

The purpose of this section is to examine the effect of a finite rather than infinite time interval. In the first part, we examine the error that arises when a time series is averaged over a finite interval. We will then apply the results to the case of determining the amplitude of a harmonic signal in the presence of equally-spaced sidebands. Proper choice of the averaging time is shown to lead to the complete elimination of error.

1. Averaging Over Finite Time Intervals

Consider any time series F(t). This can be represented by the Fourier expansion

$$F(t) = \sum_f \left[G(f)\cos(2\pi ft) + H(f)\sin(2\pi ft) \right] , \tag{3.3}$$

where f is the frequency. The dc component of F(t) can be analytically obtained by averaging over an infinite time:

$$G(0) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} F(t)\,dt . \tag{3.4}$$

For *finite* values of the time T, the average only yields an approximate value of the dc component. The general finite-time average is

$$\overline{F} = \frac{1}{T} \int_{T_o}^{T+T_o} F(t)\,dt , \tag{3.5}$$

where T_o is arbitrary. Substituting the Fourier expansion (3.3) into the average (3.5), and performing the integration, yields

$$\overline{F} = \sum_f \left[\frac{\sin[2\pi f(T+T_o)] - \sin(2\pi fT_o)}{2\pi fT} G(f) \right.$$

(3.6)

$$- \frac{\cos[2\pi f(T + T_o)] - \cos(2\pi f T_o)}{2\pi f T} H(f) \Bigg] .$$

In the limit $T \to \infty$, note that Eq. (3.6) correctly reduces to G(0). Eq. (3.6) shows that the effect of averaging over a finite time interval is to *low-pass filter* the time series. The filtering depends upon the phase of the time window compared to the time series. For example, for an average over $-T/2$ to $T/2$ (i.e., $T_o = -T/2$), the filter applied to G(f) is $\sin(\pi f T)/(\pi f T)$, and the filter applied to H(f) is zero. This is in fact the simplest case with which to deal, but will continue with our general choice (3.5) in order to ensure the generality of the results.

In Eq. (3.6), note that the filters simultaneously vanish if $2\pi f T = 2\pi, 4\pi, 6\pi, \ldots$, except for the G(f) filter at f = 0. Hence, if the Fourier components of the time series are restricted to the equally-spaced values $f = 1/T, 2/T, 3/T, \ldots$, *no error results in the finite-time average*; i.e., the value of Eq. (3.5) is G(0). The reason for this is simply that the integration interval T in Eq. (3.6) corresponds to integral numbers of cycles of these frequencies, so they will all average to zero.

2. Determination of the Amplitude of a Signal in the Presence of Discrete Noise

We now consider the time series

$$F(t) = A\cos(2\pi ft + \gamma) + B\cos[2\pi(f \pm \Delta f)t + \varphi] . \qquad (3.7)$$

The first term is considered to be the "signal," and the second term the "noise," where $\Delta f > 0$. The problem is to determine the signal amplitude A from F(t) alone. Analytically, this can be accomplished by down-converting the signal frequency f to dc, and then averaging:

$$A_c = \lim_{T\to\infty} \frac{1}{T} \int_{-T}^{T} F(t)\cos(2\pi ft)\,dt \ , \qquad (3.8a)$$

$$A_s = \lim_{T\to\infty} \frac{1}{T} \int_{-T}^{T} F(t)\sin(2\pi ft)\,dt \ , \qquad (3.8b)$$

$$A = \left(A_c + A_s\right)^{1/2} . \qquad (3.8c)$$

Compared to the dc case (3.4), a factor of 2 has been included in Eqs. (3.8a,b) because half of the amplitude at frequency f is converted to zero frequency. (The other half is converted to frequency 2f.)

As examined in Sec. 3.2A, the integrations (3.8a,b) will in general only be approximate for a finite averaging time T. We thus consider the now-approximate quantities

$$A_c = \frac{2}{T} \int_{T_o}^{T+T_o} F(t)\cos(2\pi ft)\,dt \ , \qquad (3.9a)$$

$$A_s = \frac{2}{T} \int_{T_o}^{T+T_o} F(t)\sin(2\pi ft)\,dt \ , \qquad (3.9b)$$

where T_o is arbitrary.

The first problem is to ensure that the results are exact when B = 0 in Eq. (3.7). This is readily seen to occur if the averaging time is an integral number of cycles of the signal:

$$T = \frac{n}{f}, \qquad n = 1, 2, 3, \ldots \,. \tag{3.10}$$

As shown in Sec. 3.2A, averaging over a finite time interval has the effect of introducing a low-pass filter that yields no error if the frequency spacing of the unwanted Fourier components is 1/T from zero frequency. Due to the down-converting in Eqs. (3.9a,b), this corresponds here to unwanted Fourier components with spacing 1/T about the frequency f of the signal. That is, there is no error due to a component of frequency $f \pm m\Delta f$, where $m = 1, 2, 3, \ldots$, if

$$T = \frac{1}{\Delta f} \,. \tag{3.11}$$

Combining Eqs. (3.10) and (3.11), we find that all error will be eliminated if

$$n = \frac{f}{\Delta f}, \qquad n = 1, 2, 3, \ldots \tag{3.12}$$

In the implementation of a DFT, the integration time n/f and the frequencies f and Δf must be chosen such that the condition (3.12) holds. In practice, the frequencies are usually chosen to be convenient numbers, so that their ratio is automatically an integer. The result (3.12) is explicitly verified in the next section.

3. Explicit Calculation of the Error Due to Finite-Time Averaging of the Noise

From Eqs. (3.9a,b), the errors that arise due to the presence of the noise in Eq. (3.7) are

$$\Delta A_c = \frac{2B}{T}\int_{T_o}^{T+T_o} \cos[2\pi(f \pm \Delta f)t + \varphi]\cos(2\pi f t)\,dt \;, \tag{3.13a}$$

$$= \frac{2B}{T}\left\{\cos(\varphi)\int_{T_o}^{T+T_o} \cos[2\pi(f \pm \Delta f)t]\cos(2\pi f t)\,dt \;-\; \sin(\varphi)\int_{T_o}^{T+T_o} \sin[2\pi(f \pm \Delta f)t]\cos(2\pi f t)\,dt\right\} ,$$

$$\Delta A_s = \frac{2B}{T}\int_{T_o}^{T+T_o} \cos[2\pi(f \pm \Delta f)t + \varphi]\sin(2\pi f t)\,dt \;, \tag{3.13b}$$

$$= \frac{2B}{T}\left\{\cos(\varphi)\int_{T_o}^{T+T_o} \cos[2\pi(f \pm \Delta f)t]\sin(2\pi f t)\,dt \;-\; \sin(\varphi)\int_{T_o}^{T+T_o} \sin[2\pi(f \pm \Delta f)t]\sin(2\pi f t)\,dt\right\} .$$

Using the identities cos(x)cos(y) = [cos(x+y)+cos(x–y)]/2, sin(x)cos(y) = [sin(x+y)+sin(x–y)]/2, and sin(x)sin(y) = [–cos(x+y)+cos(x–y)]/2, we express Eqs. (3.13a,b) as

$$\Delta A_c = \frac{B}{T}\cos(\varphi)\int_{T_o}^{T+T_o}\left\{\cos[2\pi(2f \pm \Delta f)t] + \cos(2\pi\Delta f t)\right\}dt$$

$$- \frac{B}{T}\sin(\varphi)\int_{T_o}^{T+T_o}\left\{\sin[2\pi(2f \pm \Delta f)t] + \sin(2\pi\Delta f t)\right\}dt \;, \tag{3.14a}$$

$$\Delta A_s = \frac{B}{T}\cos(\varphi)\int_{T_o}^{T+T_o}\left\{\sin[2\pi(2f \pm \Delta f)t] - \sin(2\pi\Delta f t)\right\}dt \tag{3.14b}$$

$$+\ \frac{B}{T}\sin(\varphi)\int_{T_o}^{T+T_o}\left\{\cos[2\pi(2f \pm \Delta f)t] - \cos(2\pi\Delta ft)\right\}dt\ .$$

Consider the second terms in the integrands in Eqs. (3.14a,b). Due to Eq. (3.11), the integration is over one cycle of these terms, so the integration yields zero. Using Eqs. (3.10) and (3.11), we can express the period of the first terms as $1/(2f \pm \Delta f) = 1/(2n/T \pm 1/T) = T/(2n \pm 1)$. Because this integration time T is an integral number of this period, the integration yields zero. Hence, the errors vanish ($\Delta A_c = \Delta A_s = 0$), and so sidebands at $f \pm \Delta f$ do not contribute to Eqs. (3.9a,b).

Now suppose that there is another sideband at $f \pm 2\Delta f$, and that Eqs. (3.10) and (3.11) still hold. The second terms in the integrands of Eqs. (3.14a,b) still vanish, because the integrations are over two cycles. The period of the first terms is now $T/[2(n \pm 1)]$, which still leads to vanishing integrals because the integration time T is an integral number of this period. The argument generalizes to sidebands at $f \pm m\Delta f$, where $m = 1, 2, 3, \ldots$.

C. TESTS OF THE NUMERICAL PROGRAM

To test the numerical program, we consider two simple cases. One is the propagation of a pure tone and the other is the suppression of a weak signal of high frequency by a high amplitude pump of low frequency in the preshock region. In both cases, the numerical results are in excellent agreement with analytical theories.

1. Pure Tone Propagation

Consider the pure tone

$$u(0,t) = u_0 \sin(2\pi ft) \ , \tag{3.15}$$

at $x = 0$. Due to nonlinear effects, energy flows to harmonics of the fundamental frequency which therefore grow as a function of distance. In the preshock region, the energy in the higher harmonics equals the energy loss at the main frequency. Consequently, the amplitude at frequency f decreases with distance. From Eq. (2.10), in dimensionless variables ($c = \beta = 1$) the amplitude as a function of distance is

$$B_1 = \frac{2J_1(2\pi fu_0x)}{2\pi fu_0x} \ . \tag{3.16}$$

According to the shock distance (2.11), Eq. (3.16) is valid only for distances $x < 1/2\pi fu_0$, and the amplitude thus decreases monotonically.

Figure 3.1 shows a comparison between the numerical integration and theory, for a signal with amplitude $u_0 = 10^{-4}$ and frequency $f = 50$. The shock inception distance equals 31.83 units in this case. For 20 time steps per cycle there is a small but significant error. Increasing the number of time steps to 200 gives excellent agreement with theory.

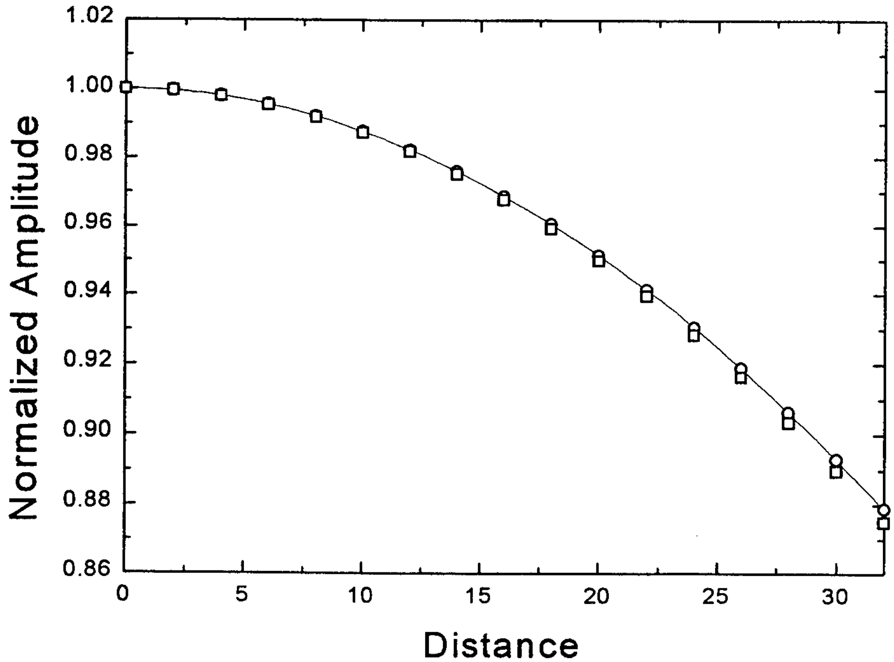

Fig 3.1 Normalized amplitude as a function of distance for a pure tone injected at the origin. The amplitude of the pure tone is 10^{-4} and the frequency is 50. The points are the result of numerical integration, and the solid line corresponds to the theoretical expression (3.16). The squares correspond to 20 time steps per cycle, and the circles correspond to 200 time steps per cycle.

2. Suppression of a Signal by a Pump

As discussed in Ch. II. and Sec. D., the suppression of a signal by a pump is formulated in terms of the boundary condition (2.12) at $x = 0$

$$u(0,t) = u_p \sin(2\pi f_p t) + u_s \sin(2\pi f_s t) , \tag{3.17}$$

where the pump frequency is $f_p = 1$ and the signal frequency is $f_s = 50$. The weak signal has amplitude 10^{-4} and has little effect on the propagation of the pump with amplitude 10^{-2}. However, the pump will modulate the signal and generate sidebands at frequencies $f_s \pm f_p$, thereby removing energy from the signal. In dimensionless variables, Fenlon's approximate solution (2.13) for the evolution of the signal is

$$u(x,t) = u_s J_o\left(2\pi f_s u_p x\right) \sin\left[2\pi f_s (t - x/c)\right] . \tag{3.18}$$

Figure 3.2 shows a comparison between the numerical integration and the theory (3.18). The shock inception distance (2.11) for the pump equals 15.92 units. The number of time steps per cycle of the signal is 20. There is excellent agreement between theory and numerical integration.

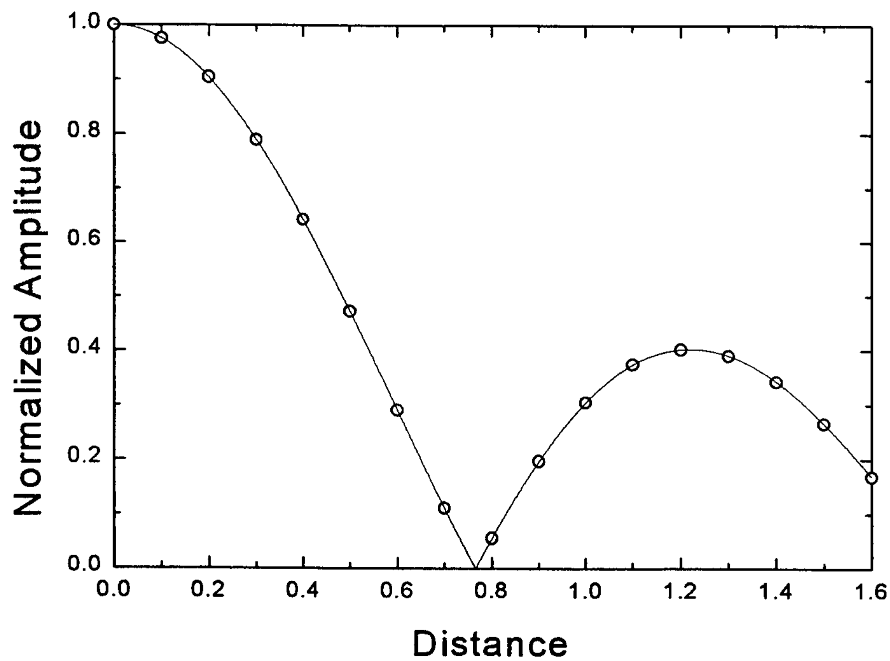

Fig 3.2 Suppression of a signal with frequency 50 by a pump of amplitude 10^{-2} and frequency 1. The signal's amplitude is normalized to its value of 10^{-4} at the origin. The points are the result of numerical integration, and the solid line corresponds to the theoretical expression (3.18). The number of time steps per cycle of the signal is 20.

IV. NUMERICAL SIMULATIONS OF ABSORPTION OF SOUND BY NOISE

In this chapter we present numerical simulations of the absorption of sound by noise in one dimension. We verify the Gaussian attenuation of a signal in the presence of noise and present two new results. In the first, we investigate the transition from Gaussian to Bessel dependence as a function of resolution in the detection of a signal. In the second result, as a test of the breakdown of translational invariance, we investigate the evolution of the amplitude of a signal injected downstream from the noise.

A. ABSORPTION BY NOISE

As stated in Ch. I. and Sec. E., a small-amplitude signal of frequency f interacting with finite-amplitude broadband noise is predicted to attenuate in distance as a Gaussian. We choose the signal to be injected with peak particle velocity u_s at x = 0, so the particle velocity of the signal at x = 0 is

$$u_{signal}(0,t) = u_s \cos(2\pi f t) \ . \tag{4.1}$$

The rms (root-mean-square) amplitude of the particle velocity of the signal as a function of position is predicted to attenuate as

$$V_{signal} = \frac{u_s}{\sqrt{2}} \exp\left(-\Gamma x^2\right) , \tag{4.2}$$

where the Gaussian coefficient (2.14) is, in dimensionless units,

$$\Gamma = 2\pi^2 f^2 u_{rms}^2 , \tag{4.3}$$

where u_{rms} is the dimensionless rms particle velocity of the noise, which is constant in space (although the spectral components of noise change).

The noise is described at $x = 0$ by the particle velocity

$$u_{noise}(0,t) = \frac{u_0}{\sqrt{N}} \sum_{n=1}^{N} \cos\left[2\pi\left(1+\frac{n-1}{N}\right)t + \varphi_n\right] . \tag{4.4}$$

where φ_n are random phases. Note that the total average acoustic intensity of the noise (4.4) is

$$u_{rms}^2 = \frac{u_0^2}{2} . \tag{4.5}$$

The frequency components of the noise (4.4) are equally spaced in a band of frequencies between 1 and 2. In the preshock region, the theory should be accurate (i.e., the signal should attenuate as the predicted Gaussian) for a sufficiently large number N of components of the noise. If all the phases in Eq. (4.4) are set to $\pi/2$, the noise shock inception distance is (Fenlon, 1970)

$$L = \frac{\sqrt{N}}{\pi u_0 (3N-1)} . \tag{4.6}$$

For amplitude $u_0 = 0.01$ and N = 50 components, the shock inception distance (4.6) is approximately L = 1.51 units. According to Eq. (2.7) random phases would yield a larger shock inception distance because Eq. (4.4) with all the phases set to $\pi/2$ has the maximum slope at the origin.

Figure 4.1 shows a comparison between the numerical integration and theory, for a signal with amplitude $u_s = 10^{-4}$ and frequency $f = 50$. In the numerical integration, there are 20 time steps per cycle of the signal. Note that the amplitude of the signal appears to asymptote to a nonzero constant value. Increasing the averaging time does not improve this condition. Also, for the parameters of the problem, we numerically determined the shock inception distance to be greater than 1.4 units, so shocking is not responsible for the departure from Gaussian attenuation. A possible explanation for the deviation may be that the noise (4.4) has developed strong correlations. This is a subject of future investigations.

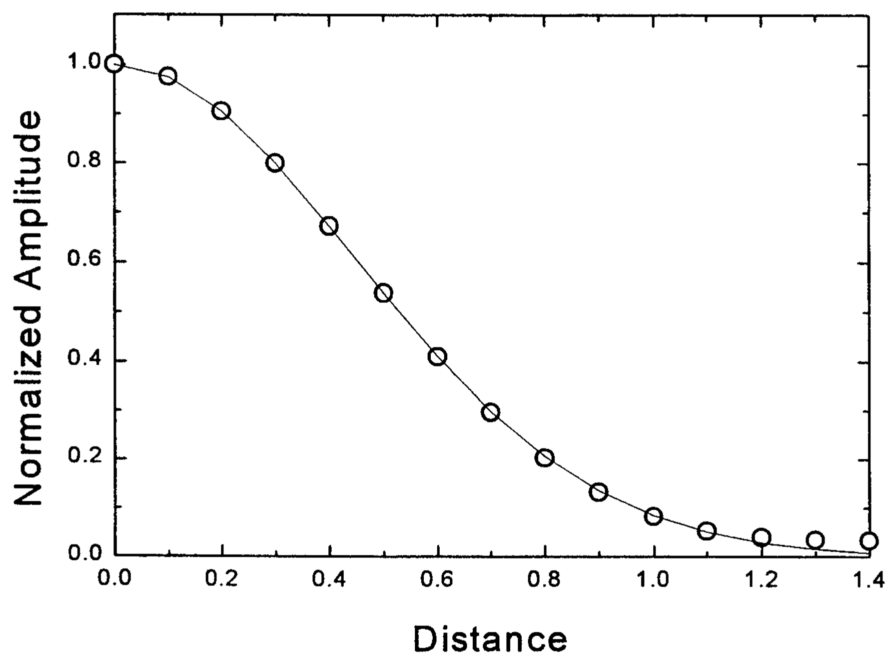

Fig 4.1 Attenuation of a signal with frequency 50 as a function of distance due to the noise (4.4) with amplitude $u_0 = 0.01$. The points are the result of numerical integration with 20 time steps per cycle of the signal. The curve corresponds to theory.

B. RESOLUTION OF DETECTION AND GAUSSIAN-TO-BESSEL TRANSITION

The result obtained above depends on the resolution of the detection. If the averaging time is less than the period of the lowest frequencies, poor resolution will result in fundamentally different behavior. Rather than the monotonic decrease on the amplitude, the signal can exhibit restitution effects. For discrete noise with equally spaced frequency components, therefore, poor resolution restores reversibility.

The resolution of the detection explains a question that naturally arises when it is noted that Rudenko's Gaussian prediction does not depend upon the bandwidth of the noise, but only upon the total average intensity. If the bandwidth is reduced while the intensity is held constant, the amplitude of a signal must continue to attenuate as the same Gaussian. In the limit when the bandwidth is zero, however, the amplitude of a signal must have a Bessel-function dependence (Ch. II, Sec. E and Ch. III, Sec. C-2). How is the transition from Gaussian to Bessel dependence made? In principle, as long as the bandwidth is not exactly zero, the signal will attenuate as the Gaussian, although this will only be apparent for averaging times that are substantially greater than the inverse of the bandwidth. In practice, the resolution of the detection will eventually be insufficient as the bandwidth is reduced. The signal will then effectively be in the presence of a pure tone, and the Bessel dependence of the signal's amplitude will be observed. This is a striking situation in which the time reversibility of a system depends upon the observer. That is, the fundamental property of time reversibility can only be established if the overall system of the waves and the observer is considered.

Figure 4.2 shows the attenuation of a signal with frequency 50 due to the noise

$$u_{\text{noise}}(0,t) = \frac{u_0}{\sqrt{N}} \sum_{n=1}^{N} \cos\left[2\pi\left(1+\frac{n-1}{8N}\right)t + \varphi_n\right], \tag{4.7}$$

for two different resolutions. The band of frequencies is between 1.00 and 1.25, with $N = 50$ components and amplitude $u_0 = 0.01$. The results were obtained with 20 time steps per cycle of the signal.

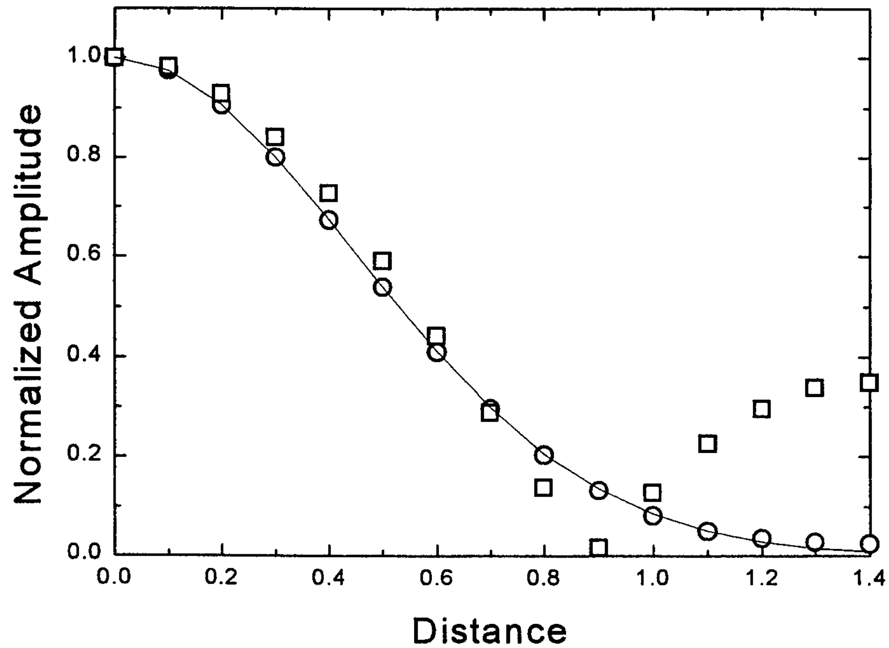

Fig 4.2 Attenuation of a signal with frequency 50 as a function of distance due to the noise (4.7) with amplitude $u_0 = 0.01$. The circles are the result of numerical integration with 20 time steps per cycle of the signal and averaging over a time of 400 (one period of the step frequency 0.0025). The squares correspond to averaging over a time of 1 (one period of the lowest frequency 1 of the band). The curve corresponds to the Gaussian attenuation theory.

The results of high resolution are represented by circles. In this case, the averaging time is 400. This time equals a period of the step frequency 0.0025, which is the smallest frequency resulting from nonlinear interactions. The curve represents the Gaussian attenuation theory. There is good agreement because the sidebands, which result from different components of the noise interacting with both themselves and the signal, are all averaged out (refer to Sec. B).

The squares correspond to averaging over a time equal to one period of the lowest frequency of the band. This is the low resolution limit. The restitution effects of the signal are apparent. The location of the zero depends on the effective intensity of the noise when there is a lack of resolution of the components of the noise, and thus may depend on the phases of the components. Future investigations will address this issue.

C. DOWNSTREAM INJECTION OF A SIGNAL

As a test of the breakdown of translational invariance we consider the evolution of a signal that is injected downstream relative to the noise at the origin. The signal is expected to attenuate monotonically as a Gaussian. This is suggested by the independence on the spectral shape of the noise in Rudenko's solution where the attenuation coefficient (4.3) depends only on the total energy of the noise. As the noise propagates downstream, nonlinearities modify the spectrum but the total energy of the noise remains the same.

On the other hand, the main assumption in Rudenko's theory is that the noise is normally distributed at the origin. As the noise propagates downstream it may develop correlations due to nonlinearities. Deviations from the expected Gaussian attenuation of a signal injected downstream may thus serve as a probe of the degree of correlations of the noise.

Figure 4.3 shows the evolution of the amplitude of a signal injected downstream at $x = 0.3$ units for the same parameters used in Sec. 4.1 and Fig. 4.1. We have chosen the initial amplitude of the signal injected at the downstream point to be equal to the amplitude that the signal of Fig 4.1 injected at the origin would have at that point. The solid line corresponds to the predicted Gaussian with the downstream injection point as the origin. Also plotted in the figure is the numerical and analytical attenuation of a signal injected at the origin. As with the previous results, the fact that the numerical noise is nearly normal, may be the origin for the disagreement between theory and numerical results at distances larger that 1.3 units.

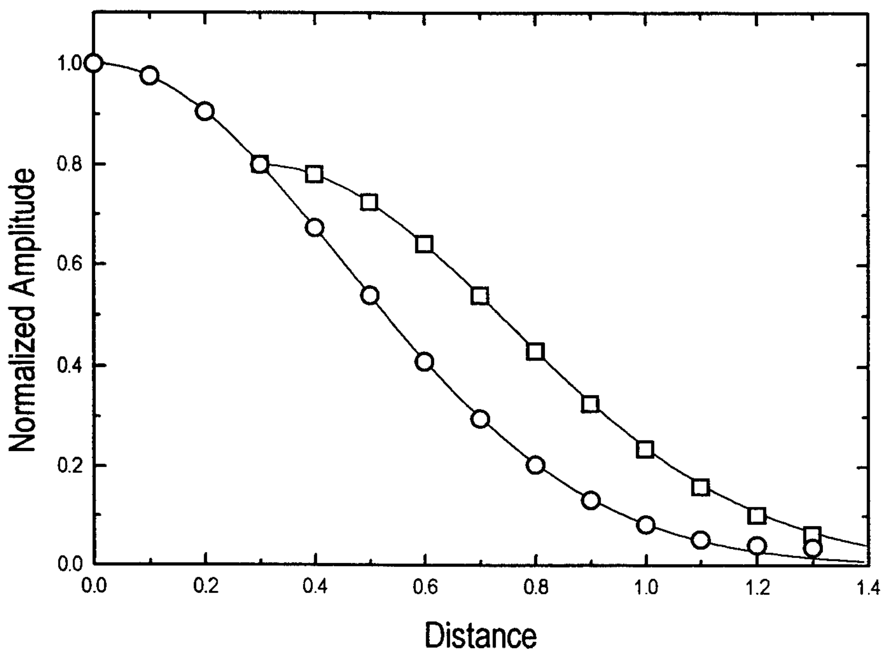

Fig 4.3 Attenuation of a signal of frequency 50 as a function of distance due to the noise (4.4) with amplitude $u_0 = 0.01$. The squares correspond to a signal injected downstream at $x = 0.3$. The circles correspond to the signal injected at the origin. The curves correspond to theory.

V. CONCLUSIONS AND FUTURE WORK

A. CONCLUSIONS

We have numerically investigated the attenuation of a monochromatic signal in the presence of discrete noise in one dimension. The predicted Gaussian attenuation has been verified by the numerical program, which is based on Riemann's implicit solution of the exact equation for the unidirectional propagation of shockless sound. The program was checked for two cases in which analytic solutions exist: propagation of a pure tone and the suppression of signal by a pump wave.

We also presented two new results. In the first, we observed a transition from Gaussian to Bessel dependence as a function of resolution in the detection of a signal. This results shows that the time reversibility of a system depends upon the observer. That is, the fundamental property of time reversibility can only be established if the overall system of the waves and the observer is considered.

In the second result, we investigated the evolution of the amplitude of a signal injected downstream from the noise. We again observed the Gaussian attenuation. This result explicitly shows that the attenuation length depends on the distance the signal has traveled, thus displaying memory and breakdown of translational invariance.

B. FUTURE WORK

Throughout this work, we have argued that small but significant discrepancies between numerical results and the analytical predictions may have their origin in the fact that the numerical noise is nearly normal. In fact, different sets of values of the random phases yield slightly different results about an average value that agrees with theory except for large distances.

Future investigations should test the statistics of the numerical noise and quantify the deviations from normal distribution at the origin. Deviations from the normal distribution should also be quantified downstream where strong correlations may develop due to nonlinearities. Light may be shed onto the problem by using noise with *quasiperiodic* frequencies, where the spacing between frequency components is incommensurate. In principle, nonlinearities cause an infinite number of frequencies to be produced in this case. Hence, it may be that substantially fewer initial frequency components of the noise are required to yield an approximate Gaussian attenuation of a signal. This may be important in applications where an efficient suppression of high frequency sound is desired.

Another topic of future investigation should be stochastic quasimonochromatic noise. In this case, both the dependence on the statistics of the noise and the transition from Gaussian to Bessel dependence can be probed. Based on our observation that this transition can be induced by insufficient resolution of the detection, we believe that the slightest stochasticity of the noise will cause the signal to attenuate as a Gaussian if the detection is accurate.

APPENDIX A. FORWARD TIME NUMERICAL PROGRAM

```
/*********************************************************************/
/*          Numerical integration of Riemann's equation             */
/*           for propagation of sound in one dimension              */
/*                                                                   */
/*********************************************************************/

    /* This program simulates the propagation of a CW in the presence
    of one dimensional noise with a flat distribution.   The
    signal's amplitude is calculated as a function of distance.
    The noise is discrete and the number of components N can be
    made to vary from one to several.  Regardless of the number
    of components, the total energy of the noise is kept constant
    by dividing the peak amplitude of each noise component by the
    square root of N.

    LAST UPDATE : 21 OCTOBER 1996     */

/*      PROCESSOR DIRECTIVES        */
#include "math.h"
#include "stdio.h"
#include "stdlib.h"

/*      MACRO DEFINITIONS           */
#define pi 3.1415926535898
#define SIZE 100002

main()
{
  int     N,          /* number of noise components   */
          i,j,k,l;

  double U[SIZE],
          u[SIZE],    /* dimensionless particle velocity arrays */
          T[SIZE],    /* dimensionless time arrays */
          tau[SIZE],
          r[102],     /* random seed */
          R[203],
          R1[203],

          u0,         /*dimensionless peak velocity of one of noise
                                      components */
          us,         /*dimensionless peak velocity of one of noise
                                      components */
          f0,         /*frequency of signal */

          x,dx,       /*distance and distance step*/
          t,dt,       /*time and time step*/

          max,        /*maximum number generated from random function*/
          seed,       /*seed for random number generation*/

          d1,d2,
          a1,a2,      /*variables used for the LOCK-IN*/
          k1,k2,k3,k4;

  /* Reading the data form the input file */
```

```
  scanf("%lf      %lf   %lf   %lf   %d\n",&u0,&us,&f0,&seed,&N);


/* Variables Initialization */

  max=pow(2.0,31.0)-1.0;
  d1=d2=t=x=0.0;
  u0=u0*sqrt(1.0/N);               /*thus, the noise energy is the input
                                            u0^2 */
  dt=0.001;
  dx=0.1;



  for(k=1; k<30; k++){             /*Initialize R matrices to zero */
     R[k]=0.0;
     }

  for(l=0; l<20; l++){
     srandom(seed);
     for(i=0; i<N; i++){           /*Random phase of noise components */
        r[i]=(2.0*pi*random()/max);
        }

     for(k=1; k<20; k++){          /*Propagate up to dist of k(max)*dx*/

        for(i=0; i<SIZE; i++){     /*Time and velocity arrays
                                            initialization */
           U[i]=T[i]=0.0;
           }

        for(i=0; i<N; i++){        /*Flat noise in a band between
                                              1 and 2 */
           for(j=0; j<SIZE; j++){
              T[j]=j*dt;
              u[j]=u0*sin(2.0*pi*(1.0+(double)(i)
                    /(double)(N))*T[j]+r[i]);
              U[j]=U[j]+u[j];
              }
           }

        for(i=0; i<SIZE; i++){     /*Boundary conditions*/
           U[i]=U[i]+us*sin(2.0*pi*f0*T[i]);
           }

        for(i=0; i<SIZE; i++){     /*Riemann's propagation time*/
           tau[i]=T[i]-x;
           tau[i]=tau[i]-x/(1.0+U[i]);
           }

        for(i=0; i<SIZE; i++){     /*Extracts the signal component only*/
           U[i]=sin(2.0*pi*f0*tau[i]);
           }

        for(i=0; i<(SIZE-2); i++){/*In-phase and quadrature components*/

           a1=U[i]; a2=U[i+1];
           t=i*dt;
```

```
          k1=cos(2.0*pi*f0*t);
          k2=cos(2.0*pi*f0*(t+dt));
          k3=sin(2.0*pi*f0*t);
          k4=sin(2.0*pi*f0*(t+dt));

          d1=d1+(((a1*k1)+(a2*k2))*(tau[i+1]-tau[i])/2.0);
          d2=d2+(((a1*k3)+(a2*k4))*(tau[i+1]-tau[i])/2.0);

          }

       R1[k]=2.0*f0*sqrt(d1*d1+d2*d2)/(5000.0);
       x=k*dx; d1=d2=t=0.0;
       }

    seed=seed+1.0;
    for(k=1; k<20; k++){
      R[k]=R[k]+R1[k];
      }
    x=0.0;
    }

  x=0.0;
  for(k=1; k<20; k++){              /* Print the result */
    printf("%f   %.12f\n",x,R[k]/20.0);
    x=k*dx;
    }
}
```

APPENDIX B. FORWARD TIME NUMERICAL PROGRAM: DOWNSTREAM INJECTION

```
/**********************************************************************/
/*          Numerical integration of Riemann's equation              */
/*           for propagation of sound in one dimension               */
/*                                                                   */
/**********************************************************************/

    /*  This program simulates the propagation of a CW in the presence
    of one dimensional noise with a flat distribution. In particular,
    this is designed to see how the absorption of signal put into the
    developed noise works.
        The signal's amplitude is calculated as a function of distance.
    The noise is discrete and the number of components N can be
    made to vary from one to several.  Regardless of the number
    of components, the total energy of the noise is kept constant
    by dividing the peak amplitude of each noise component by the
    square root of N.

    LAST UPDATE : 21 OCTOBER 1996     */

/*      PROCESSOR DIRECTIVES        */
#include "math.h"
#include "stdio.h"
#include "stdlib.h"

/*      MACRO DEFINITIONS           */
#define pi 3.14159265
#define SIZE 100002

main()
{
  int    N,          /* number of noise components   */
         i,j,k,l;

  double U[SIZE],
         u[SIZE],    /* dimensionless particle velocity arrays */
         T[SIZE],    /* dimensionless time arrays */
         tau[SIZE],
         r[102],     /* random seed */
         R[203],
         R1[203],

         u0,         /*dimensionless peak velocity of one of noise
                                components */
         us,         /*dimensionless peak velocity of one of noise
                                components */
         f0,         /*frequency of signal */

         x,dx,       /*distance and distance step*/
         t,dt,       /*time and time step*/

         max,        /*maximum number generated from random function*/
         seed,       /*seed for random number generation*/

         d1,d2,
         a1,a2,      /*variables used for the LOCK-IN*/
```

```
        k1,k2,k3,k4;

  /* Reading the data form the input file */

  scanf("%lf      %lf   %lf   %lf   %d\n",&u0,&us,&f0,&seed,&N);

  /* Variables Initialization */

  max=pow(2.0,31.0)-1.0;
  d1=d2=t=x=0.0;
  u0=u0*sqrt(1.0/N);             /*thus, the noise energy is the input
                                         u0^2 */
  dt=0.001;
  dx=0.1;
  us=us*exp(-(2.0*pi*50.0*u0*0.3)*(2.0*pi*50.0*u0*0.3)*N/4.0);

  for(k=1; k<30; k++){           /*Initialize R matrices to zero*/
     R[k]=0.0;
     }

  for(l=0; l<20; l++){
     srandom(seed);
     for(i=0; i<N; i++){         /*Random phase of noise components*/
        r[i]=(2.0*pi*random()/max);
        }

     for(k=3; k<20; k++){        /*Propagate up to dist of k(max)*dx*/

/**********************************************************************/
/*          Propagate the noise componets up to 0.3m first             */
/**********************************************************************/

        x=0.3;
        for(i=0; i<SIZE; i++){   /*Time and velocity arrays
                                    initialization */
           U[i]=T[i]=0.0;
           }

        for(i=0; i<N; i++){      /*Flat noise in a band between
                                            1 and 2*/
           for(j=0; j<SIZE; j++){
              T[j]=j*dt;
              u[j]=u0*cos(2.0*pi*(1.0+(double)(i)/
                   (double)(N))*T[j]+r[i]);
              U[j]=U[j]+u[j];
              }
           }

        for(i=0; i<SIZE; i++){   /*Riemann's propagation time for
                                             noise*/
           T[i]=T[i]-x/(1.0+U[i]);
           }

/**********************************************************************/
/*      Add signal to developed noise and propagate both of them       */
/*                        up to k(max)*dx                              */
/**********************************************************************/
```

```
        x=k*dx;
        for(i=0; i<SIZE; i++){   /*Inject the signal */
           U[i]=U[i]+us*cos(2.0*pi*f0*T[i]);
           }

        for(i=0; i<SIZE; i++){   /*Riemann's propagation time for signal
                                             and noise*/
           tau[i]=T[i];
           tau[i]=tau[i]-(x-0.3)/(1.0+U[i]);
           }


        for(i=0; i<SIZE; i++){   /*Extracts the signal component only*/
           U[i]=us*cos(2.0*pi*f0*T[i]);
           }

        for(i=0; i<SIZE-2; i++){ /*In-phase and quadrature components*/

           a1=U[i]; a2=U[i+1];
           t=i*dt;

           k1=cos(2.0*pi*f0*tau[i]);
           k2=cos(2.0*pi*f0*(t+dt));
           k3=sin(2.0*pi*f0*tau[i]);
           k4=sin(2.0*pi*f0*(t+dt));

           d1=d1+(a1*k1)*(tau[i+1]-tau[i]);
           d2=d2+(a1*k3)*(tau[i+1]-tau[i]);

           }

        R1[k]=2.0*f0*sqrt(d1*d1+d2*d2)/(5000.0*0.0001);
        x=k*dx; d1=d2=t=0.0;
        }

     seed=seed+1.0;
     for(k=3; k<20; k++){
       R[k]=R[k]+R1[k];
       }
     }

  for(k=3; k<20; k++){            /*Print the result*/
     x=k*dx;
     printf("%f   %.12f\n",x,R[k]/20.0);

     }
}
```

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